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Considerations on Computing the Fractal Dimension of Financial Time Series

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Abstract

In the last decades, fractal geometry imposed itself in various fields, playing an important role in financial analysis. Fractal properties of financial instruments' prices, market indicators and so have generated a particular interest. Among these, fractal dimension is associated to the volatility of financial time series. There are various methods for computing the fractal dimension, the most known is the box-counting algorithm based on Hausdorff dimension. The algorithm provides the box-counting dimension which offers a good description about the complexity of the form. Sometimes though, in financial applications, a "global" image is not enough: a financial time series can have regions with different roughness degrees: the price of an asset can be stable over a period of time, while it can register high volatility over the next period. Therefore, another approach is proposed: the local fractal dimension. Both algorithms are implemented into an original software application developed by the author.

Keywords: fractal analysis, fractal dimension, box-counting dimension, financial volatility

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1. Introduction

Fractals started to draw researchers' attention since the end of the 19th century and beginning of the 20th century, when the illustrious mathematicians like Giuseppe Peano, Niels Fabian Helge von Koch, Wacław Franciszek Sierpiński, and so began to imagine fractals, attracted by their strange properties. However, fractal geometry was initiated much later, around 1970, by the French mathematician Benoit Mandelbrot. He introduced the notion of a "fractal" in order to describe an irregular geometric object with infinite details at any scale representation.

Using the new branch of mathematics, Mandelbrot was trying to explain natural phenomena, emphasizing the presence of fractals anywhere in the surrounding nature, in human pursuits (music, painting, architecture) [8]. Soon, fractal geometry has proven to be a very valuable instrument in various fields. In image processing, the success of fractal techniques was outstanding [4][5]. Significant applications of fractal geometry can be found in financial field.

Fractal properties of the financial time series have generated a particular interest. It has been noticed that, viewed from a 'distance' (i.e. viewed over a long time scale) financial time series appear smooth in general, almost deterministic. But at shorter time scales, the irregularities of the series increase, yielding to a more complex structure (detail at any scale). Moreover, the detailed structure is somehow similar to the whole structure (self-similarity) [1][9][10].

It is also known that the fractal dimension is a measure of the roughness or complexity of a form. The more complex the form is, the greater its fractal dimension will be [3]. Fractal dimension can be associated to the volatility of a financial instrument: it measures how much and how quickly the value of an investment, market, or market sector changes.

Volatility does not measure the direction of asset changes. Considering two financial instruments, the one with higher volatility will have larger swings in values over a given period of time, yielding a greater fractal dimension [13].

In the last decades, financial platforms have integrated various fractal techniques to help analysts' decisions [7][15]. Fractal properties, as well as fractal dimension, are helpful in understanding the market and allows some investors to predict financial risk.

This paper is structured as follows: the first section is dedicated to the Introduction, the second section presents the fractal dimension: the Hausdorff dimension and the self-similar dimension, and the box-counting algorithm. In the third section an algorithm for computing the local fractal dimension is described. The algorithms were applied on financial instruments in order to measure their volatility. The paper is ended with Conclusions and Bibliography.

2. Fractal dimension

The Hausdorff dimension

A defining feature of fractal objects is that they are self-similar, meaning that closer we get, more details similar to the hole image we observe. An immediate effect is that the size of a fractal object depends on the unit used to measure it: the smaller unit will cover more details of the fractal, leading to a higher dimension. This strange phenomenon, known as the *coastline paradox*, was first noticed by the English mathematician Lewis Fry Richardson: trying to find out the length of the border between Spain and Portugal, he consulted the encyclopaedias of both countries and discovered they published two different lengths: according to the Spanish encyclopaedia the border was 987 km length, while the Portuguese encyclopaedia published the length of 1214 km. The explanation for the strange phenomenon consists in using two different units of measure. While the unit size is continuously decreased, the measure will increase continuously, yielding to an infinite measure.

The dependence of objects measure on the scale used makes them difficult to evaluate in the context of classical geometry. Their physical properties (length, area, volume) depend on the resolution. Measuring the coastline into a one-dimensional Euclidian space was not enough, the need of defining a new space dimension, non-integral, has been reached.

Several attempts of measuring the size of a fractal have been done. The most widely accepted approach has been suggested by the German mathematician Felix Hausdorff around the beginning of the second decade of the 20 century. Hausdorff defines a new concept of topological spaces, suggesting that the fractal dimension is proportional to the minimum number of spheres (cubes) of a certain size required to cover the measured object.

In general, the relationship $N(s) \approx 1/s^{DE}$ is verified, where N(s) is the number of squares, s is the square' size and DE is the Euclidian dimension of the object.

Further, in his approach, Hausdorff states that the fractal dimension of an object can be described as a fractional number $D_H \approx \frac{\log(N(s))}{\log(1/s)}$ [1]

Although widely accepted, the Hausdorff dimension is difficult to calculate. In practice, the fractal dimension is estimated using self-similarity property or using various computer algorithms; the most known algorithm is the box-counting dimension.

The dimension of self-similar fractals

A key feature of fractals is the self-similarity. Mandelbrot defined fractals as objects consisting of copies of themselves at different scales. Fractal dimension of self-similar fractals is computed using the formula:

$$D_f = \frac{\log(\text{number of self - similar copies})}{\log(\text{magnification})}$$

[2]

Self-similar fractals are very useful in testing and calibrating algorithms.

The box-counting algorithm

The box-counting algorithm is based on the Hausdorff dimension. It is expected that, for smaller s, the approximation of formula [1] to be better:

$$D = \lim_{s \longrightarrow 0} \log \frac{N(s)}{\log(1/s)} \cdot$$

[3]

If this limit exists, it is called box-counting dimension of the measured object. However, in practice, this limit converges slowly, so an alternative methods are needed. Since

$$\log(N(s)) = D * \log(1/s)$$

[4]

is the equation of a straight line described by the points (log(N(s), log(1/s)), the slope D (the fractal dimension) can be found using linear regression (least squares method).

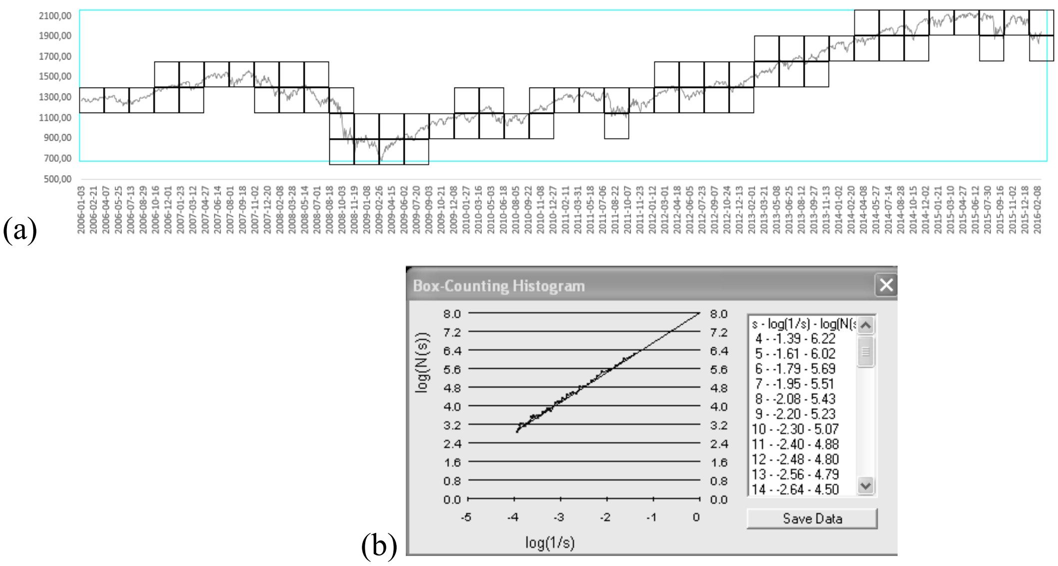
So, the box counting method consists in dividing the image into successive 4, 16, 64, so. equal squares and counting each time the number of squares which cover the object. The points of coordinates (log (N (s), log (1 / s)) where s is the common size of the squares, and N (s) is the number of squares containing the fractal are plotted. The slope of this curve, known as the *log-log curve*, will be the box-counting dimension. In order to compute the slope, the least square method will be used.

In our approach we have used an adjusted algorithm which repeatedly covers the image with squares of various increasing sizes [2]. The maximum size doesn't exceed the size of the object and the minimum value was chosen to be able to capture the finest detail of the object. The next example shows S&P500 indicator between Jan. 2006 and Feb 2016 covered with equal squares of side 30 (figure 1a) and the log-log curve (figure 1b).

Fig. 1. The S&P500 indicator between Jan. 2006 and Feb 2016.

(a) Object coverage with equal squares of side 30

(b) The slope of the log-log curve yields to the fractal dimension of 1.26



Source: processing and representation made by authors using data published in [14]

3. Algorithm for estimating the local fractal dimension

The box-counting fractal dimension offers an image on the global complexity of a shape. But often, a shape has regions with different structures. For instance, an asset can have different evolutions in time in terms of stability: some periods may be characterised by a smooth evolution, with a lower fractal dimension, while some other periods may be very volatile and have higher fractal dimension.

Fig. 2. Two periods for the S&P500 indicator with different fractal dimensions:

1.19 (1995-1999) and 1.32 (2011-2016)

Source: processing and representation made by authors using data published in [14]

The existence within an image of regions with different patterns requires the use of alternative methods, which will extend the concept of fractal to multifractals. Inspired by the Indian pattern Tara Mandala, the Italian researcher Gabriel Landini proposed a new approach meant to characterize the complexity of a local structure, through a local fractal dimension [6]. Landini successfully used his method in medicine, but the applications have significantly widened.

The local dimension is associated to any point of the shape and consists in covering the shape with an increasing vicinity of the point. The procedure is similar to box-counting method: the log-log curve based on these vicinities will then be traced and the local fractal dimension will be computed using the least squares method. The maximum size of the vicinities used for covering is constant.

The algorithm has five steps [6]:

1. Consider the current point P;

2. Consider a vicinity V of P, let the size of the vicinity to be 's';

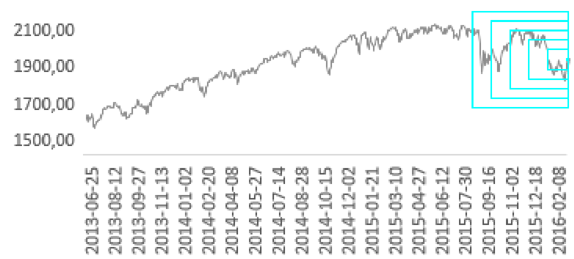
3. Count all points connected with P within the vicinity V, let this number be N(s). Any point is surrounded by 8 adjacent neighbours (N, NW, W, SW, S, SE, E, NE);

4. Repeat step 2 considering an increased vicinity, until the max_size is reached;

5. Apply the least squares method for the log-log curve obtained by plotting the points (log (N (s), log (s)). The slope is the local fractal dimension and it is associated to the current point P.

The Landini algorithm works with vicinities centred in the current point; in our approach we used the vicinities from the left side of the point, in order to measure the complexity of the asset evolution prior the current moment.

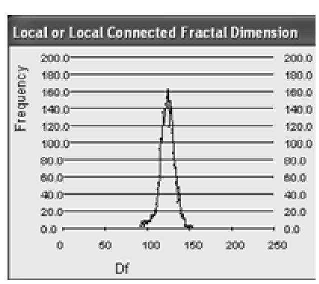
Fig. 3. The local fractal dimension of the S&P500 indicator on 08.02.2016 is 1.37



Source: processing and representation made by authors using data published in [14]

The most predominant local dimension in a region will be close to the global dimension of that region. The S&P500 local dimensions between 2011 and 2016 concentrates around the box-counting dimension of that region (1.32).

Fig. 4. The histogram of the local fractal dimensions for S&P500 between 2011 and 2016 - the values concentrates around the global dimension of 1.32



Source: processing and representation made by authors

4. Conclusions

It is already verified that fractal geometry has offered a new way of understanding the evolution in time of financial instruments' value. Various techniques have been tested, some successful, some subjected to improvements. Fractal dimension, as a measure of objects' complexity has imposed as a way of describing volatility of financial time series.

In this paper we have described a classical method for computing the fractal dimension of a form: the box-counting dimension based on the Hausdorff dimension. Since the box-counting algorithm provides an "overall" image of the volatility of the financial instruments, a new approach is proposed: the local fractal dimension, in order to characterize local evolutions, this way being a better mapping on different periods with different evolution (stable or characterized by high volatility).

Both algorithms have been tested on self-similar fractals (Sierpinski's triangle, Koch's snowflake, so), the computed fractal dimension being close to the analytical dimension (errors under 0.1). For financial series, the results have been close to those published in the scientific literature. The outputs have been extracted from an original software application developed by D.A. Crisan and presented in detail in [5].

The results are encouraging, the local dimension providing a more reliable information in analysts' decision in financial domain.

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