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VaR Analysis for the Shanghai Stock Market[1]

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Abstract

In this paper we investigated the relevance of the skewed Student's t distribution innovation in capturing long-memory and asymmetry features in the volatility of Shanghai stock markets. We also examined the performance of in-sample and out-of-sample value-at-risk (VaR) analyses using the FIAPARCH model with the normal, Student's t, and skewed Student's t distribution innovations. We found that risk managers and portfolio investors can estimate VaR and optimal margin levels most accurately by using the skewed Student's t FIAPARCH VaR models of long and short trading positions in the Shanghai stock market.

Keywords: Asymmetry, Forecasting accuracy, Long-memory, Skewed Student t-distribution, Volatility

1. Introduction

This article considers the relevance of the skewed Student's *t* distribution in estimating long-memory and asymmetry in the volatility of Chinese stock market by employing the FIAPARCH model. To further enhance the robustness of the estimation results, we compare the performance of the various VaR models with the normal, Student's *t*, and skewed Student's *t* distribution innovations.

The contribution of this article is twofold. First, we estimate VaR models with long-memory and asymmetry features in the volatility of Chinese stock market. Little work has been done on modeling the volatility of the Chinese stock market. Due to its rapid economic growth, the Chinese stock market has become the largest recipient of external investment and a hub market in the Asian region. Understanding the volatility features of the Chinese stock market is an important ingredient in measuring and controlling VaR for risk managers, portfolio investors, and regulators.

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The second contribution is that VaR estimation indicates the relevance of asymmetry and tail fatness in the return distribution of Chinese stock returns. For instance, we will show that the models with a skewed Student's *t* distribution can provide more accurate volatility forecasting results than can normal and Student's *t* distribution VaR models for both long and short positions. Consequently, our VaR analyses emphasize that the assumption of fat-tailed and asymmetric feature distribution is appropriate for evaluating the accuracy of VaR estimates in the Chinese stock market.

2. Methodology

2.1 Long-memory volatility models

In general, volatility often displays the long-memory property in which the autocorrelations of the absolute and squared returns are characterized by very slow decay over long periods. To accommodate this phenomenon, Baillie, Bollerslev and Mikkelsen (1996) extended the standard GARCH model by introducing a fractionally integrated process resulting in the FIGARCH model. Unlike the knife-edge distinction between $I(0)$ and $I(1)$ processes, the fractionally integrated process, $I(d)$, can distinguish between short memory and long-memory in conditional variances. The $\text{FIGARCH}(p,d,q)$ model is defined as $\sigma_t^2 = \omega\left[1-\beta(L)\right]^{-1} + \left\{1-\left[1-\beta(L)\right]^{-1}\varphi(L)(1-L)^d\right\}\varepsilon_t^2$, where ω, β, φ, and d are the parameters to be estimated, and $0 \le d \le 1$. L denotes the lag or backshift operator. Tse (1998) proposed the FIAPARCH model, which extends the FIGARCH model by adding the function $\left(|\varepsilon_t| - \gamma\varepsilon_t\right)^\delta$ of the APARCH model to capture asymmetry and long-memory features in the conditional variance. The $\text{FIAPARCH}(p,d,q)$ model is specified as $\sigma_t^\delta = \omega\left[1-\beta(L)\right]^{-1} + \left\{1-\left[1-\beta(L)\right]^{-1}\varphi(L)(1-L)^d\right\}\left(|\varepsilon_t| - \gamma\varepsilon_t\right)^\delta$, here δ, λ, and γ are the parameters of the model, and $\lambda(L) \equiv \sum_{i=1}^{\infty}\lambda_i L$.

The FIAPARCH model can express some well-known stylized facts of volatility: (a) for $0 < d < 1$, volatility displays the long-memory property; (b) when $\gamma > 0$, negative shocks give rise to higher volatility than do positive shocks, and vice versa; (c) the power term δ of returns for the predictable structure in the volatility pattern should be determined by the data; and (d) the FIAPARCH model also nests the FIGARCH model when $\delta = 2$ and $\gamma = 0$. Thus, the FIAPARCH model is superior to the FIGARCH model in theoretical point of view because it can capture asymmetry and long-memory features in the dynamics of conditional variance (Tse, 1998).

2.2 VaR models and tests

Following Giot and Laurnet (2003) and Wu and Shieh (2007), we explore and compare the performance of the FIGARCH and FIAPARCH models estimated on the assumption of three innovation distributions: normal, Student's *t*, and skewed Student's *t* distributions. Additionally, a one-step-ahead VaR was computed along with the results of the estimated volatility models and the given distribution.

The VaR for long and short trading positions can be rewritten as $VaR_{t,long} = \mu_t + skst_\alpha(\nu,k)\ \sigma_t$, and $VaR_{t,short} = \mu_t + skst_{1-\alpha}(\nu,k)\ \sigma_t$, where $skst_\alpha(\nu,k)$ is the left quantile at α% for the skewed Student's *t* distribution, and $skst_{1-\alpha}(\nu,k)$ is the right quantile at α% for the distribution.[2]

We calculate the VaR at the pre-specified significance level of α and then evaluate the performance by calculating the failure rate for both the left and right tails of the distribution in the sample return series. The failure rate is defined as the ratio of the number of times in which positive (negative) returns go beyond (below) the forecasted VaR to the sample size. Following Giot and Laurent (2003), testing the accuracy of the model is equivalent to testing the hypothesis $H_0: f = \alpha$ versus $H_1: f \neq \alpha$, where f is the failure rate; if the VaR model is correctly specified, the failure rate should be equal to the pre-specified significance level of α (Kupiec, 1995).

3. Empirical results

This study considers a market index traded on the Shanghai stock markets (hereafter, SSE) provided by the database of the New Informax. The data sets consist of the daily closing prices spanning from January 4, 2000, to March 31, 2011 (2,803 observations). All sample prices are converted into daily return series, i.e., $r_t = 100 \times \ln(P_t/P_{t-1})$ for $t = 1,2,\ldots$, where r_t is the returns at time t, P_t is the current price, and P_{t-1} is the previous day's price.

3.1 Asymmetric long-memory in volatility

We estimated a FIAPARCH model under the normal, Student's *t*, and skewed Student's *t* distribution innovations and compare the estimation results of the FIAPARCH$(1,d,1)$ model for SSE returns in Table 1. We used a set of diagnostic tests to check the relevance of residuals distributions and the values of Akaike's information criterion (AIC) to choose the best model from the normal, Student's *t*, and skewed Student's *t* models and calculate the Box–Pierce Q_s statistic to test the null hypothesis of the i.i.d. series of squared standardized residuals. Additionally, we used the LM ARCH statistic of Engle (1982) to test the presence of any remaining ARCH effect in the residuals.

[2] The value of parameter ν measures the degree of fat tails in the VaR density. If $\nu > 2$, the density has fat tails. The value of k determines the degree of asymmetry in the VaR density. If $k < 1$, the VaR for long trading positions will be larger for the same conditional variance than will the VaR for short trading positions. When $k > 1$, the opposite holds true. For more details, see Giot and Laurnet (2003).

Table 1. Estimation results from the FIAPARCH$(1,d,1)$ model

	Normal	Student's *t*	Skewed Student's *t*
μ	0.018 (0.028)	0.049* (0.022)	0.017 (0.027)
ω	3.616* (1.816)	1.552 (0.840)	1.592 (0.857)
α_1	0.177 (0.113)	0.069 (0.120)	0.074 (0.132)
d	0.421* (0.090)	0.345* (0.059)	0.345* (0.061)
β_1	0.528* (0.168)	0.360* (0.154)	0.361* (0.167)
γ	0.158* (0.070)	0.205* (0.073)	0.198* (0.072)
δ	1.848* (0.137)	2.036* (0.131)	2.060* (0.021)
v	-	4.718* (0.412)	4.746* (0.418)
k	-	-	-0.069* (0.021)
ln(L)	-5090.89	-4973.80	-4969.40
AIC	3.638754	3.555890	3.553463
$Q_s(20)$	9.036 [0.949]	8.796 [0.964]	8.765 [0.964]
ARCH(10)	0.294 [0.982]	0.342 [0.969]	0.350 [0.966]

Notes: Standard errors corresponding to parameter estimates are indicated in parentheses. ln(L) is the maximized value of the Gaussian log-likelihood. AIC is the Akaike information criterion. ARCH(10) represents the *F*-statistic of an ARCH test with a lag of 10. *P*-values are reported in brackets. * indicates rejection of the null hypothesis at the 5% significance level.

As shown in Table 1, the FIAPARCH model is able to capture the long-memory property due to the significance of long-memory parameters (d). This evidence suggests that the volatility of SSE returns appears to be a long-memory process, in contrast to the assumption of the efficient market hypothesis, which states that all available information is fully and immediately reflected in the prices.

The coefficients of asymmetric volatility (γ) of the three FIAPARCH models are positive and significant at the 5% level. This implies that the volatility of the SSE returns is asymmetric. That is, unexpected negative returns result in more volatility than do unexpected positive returns of the same magnitude. This evidence supports the negative relationship between current returns and future volatility as observed by Black (1976). Furthermore, the power term (δ) shows values of around 2, implying that a squared error term fits in the conditional variance specification.

The estimation results of the FIAPARCH$(1,d,1)$ model with the skewed Student's *t* distribution innovation show that the returns exhibit an asymmetric and fat-tailed distribution. The asymmetric parameters (k) were negative and significantly different from zero, which implies that the densities of residuals are skewed to the left side. Additionally, the values of the tail parameter (v) were statistically significant, indicating that the densities exhibit fat tails. As a result, we conclude that the skewed Student's *t* distribution models outperform the other models with the normal and Student's *t* distribution innovations in capturing the asymmetric and fat-tailed features of the SSE returns distribution.

As shown in the results of the diagnostic tests, the asymmetric skewed Student's *t* FIAPARCH model outperformed the symmetric normal and Student's *t* FIAPARCH models as reflected by the lowest value of AIC. Additionally, the Box–Pierce test statistics $Q_s(20)$ showed no serial correlation in squared standardized residuals, which implies that the models are correctly specified in taking into account the heteroscedasticity in the return series. Also, the insignificant values of the LM ARCH(10) test statistic indicate no remaining ARCH effect in the standardized residuals estimated from the models.

3.2 Empirical results for VaR estimation

We compute not only the in-sample VaR values to examine the estimated goodness-of-fit ability but also the out-of-sample VaR values to evaluate the 1-day-ahead forecasting performance of the estimated models. Under the assumption of different distribution innovations, we test the VaR values using the asymmetric long-memory FIAPARCH model at a significance level of α, which ranges from 0.05 to 0.0025, and assess their performance by computing the failure rate. If the VaR models are correctly specified, the failure rate will be equal to the pre-specified significance level of α.

Table 2. In-sample VaR estimation for the SSE index

	Short position				Long position			
	α	Failure rate	Kupiec LR	P-value	α	Failure rate	Kupiec LR	P-value
Normal FIAPARCH	0.95	0.9557	2.022	0.154	0.05	0.0556	1.835	0.175
	0.99	0.9871	2.106	0.146	0.01	0.0192	19.13**	0.000
	0.9975	0.9935	12.02**	0.001	0.0025	0.0085	25.22**	0.000
Student's t FIAPARCH	0.95	0.9511	0.072	0.787	0.05	0.0624	8.509**	0.003
	0.99	0.9917	0.967	0.325	0.01	0.0117	0.845	0.357
	0.9975	0.9982	0.639	0.423	0.0025	0.0039	1.943	0.163
Skewed Student's t FIAPARCH	0.95	0.9450	1.408	0.235	0.05	0.0571	2.850	0.091
	0.99	0.9900	0.000	0.996	0.01	0.0096	0.037	0.845
	0.9975	0.9975	0.000	0.998	0.0025	0.0028	0.135	0.712

Notes: The critical value of the Kupiec LR statistic is 3.84 at the 5% significant level. ** indicates rejection of the null hypothesis at the 5% level.

Table 3. Out-of-sample VaR estimation for the SSE index

	Short position				Long position			
	α	Failure rate	Kupiec LR	P-value	α	Failure rate	Kupiec LR	P-value
Normal FIAPARCH	0.95	0.9600	1.690	0.193	0.05	0.0573	0.812	0.367
	0.99	0.9920	0.325	0.568	0.01	0.0200	5.870	0.015
	0.9975	0.9973	0.008	0.927	0.0025	0.0093	8.227	0.004
Student's t FIAPARCH	0.95	0.9600	1.690	0.193	0.05	0.0586	1.126	0.288
	0.99	0.9973	5.753**	0.016	0.01	0.0106	0.032	0.855
	0.9975	1.0000	.NaN	1.000	0.0025	0.0040	0.571	0.449
Skewed Student's t FIAPARCH	0.95	0.9533	0.179	0.672	0.05	0.0573	0.812	0.367
	0.99	0.9960	3.529	0.060	0.01	0.0093	0.034	0.852
	0.9975	1.0000	.NaN	1.000	0.0025	0.0026	0.008	0.927

Note: See Table 2.

Tables 2 presents the in-sample VaR results calculated by the $\text{FIAPARCH}(1,d,1)$ model with the normal, Student's *t*, and skewed Student's *t* distribution innovations. According to these Tables, the normal and Student's *t* distribution innovations show poor performance for long and short trading positions. As α ranges from 0.05 to 0.0025, the failure rates significantly exceed the prescribed quantiles, and the null hypothesis $(f = \alpha)$ of the Kupiec LR test is often rejected in the normal and Student's *t* distribution innovations. This indicates that the normal and Student's *t* distribution models cannot explain the in-sample VaR values of the negatively skewed and fat-tailed distribution.

In contrast, the skewed Student's *t* FIAPARCH models significantly improve on the in-sample VaR performance for both long and short trading positions. The estimated failure rate is statistically equal to the pre-specified VaR level, and the Kupiec LR test does not reject the null hypothesis. Thus, we conclude that the skewed Student's *t* VaR models predict a critical loss more accurately than do models with the normal and Student's *t* distribution innovations.

We further assess the performance of the $\text{FIAPARCH}(1,d,1)$ model with the normal, Student's *t*, and skewed Student's *t* distribution innovations by computing out-of-sample VaR forecasts. Following the analysis procedure proposed by Wu and Shieh (2007), we use an iterative procedure in which the estimated model for the whole sample is estimated and then compared with the predicted 1-day-ahead VaR for both long and short trading positions.

Table 3 reports the empirical results for the out-of-sample VaR analysis, which are generally similar to those for the in-sample VaR analysis. As a result of their greater *P*-values, the skewed Student's *t* distribution VaR models generally provide more accurate volatility forecasting results than do the normal and Student's *t* distribution VaR models for both long and short trading positions. Thus, we conclude that the models with the skewed Student's *t* distribution innovation are preferable for estimating out-of-sample VaR.

4. Conclusions

In this study, we investigate the relevance of the skewed Student's *t* distribution innovation in capturing long-memory and asymmetry in the volatility of Shanghai stock market. We examine the performance of in-sample and out-of-sample VaR analyses using the $\text{FIAPARCH}(1,d,1)$ model with the normal, Student's *t*, and skewed Student's *t* distribution innovations.

After checking the statistical properties of the Shanghai stock index returns, we found that the probability distribution of returns is not normally distributed and that there is significant evidence of serial dependence in the squared returns of each market index. We also found that both returns distributions exhibit asymmetric and fat-tailed features. Such features motivate the use of non-normal distribution innovations.

From the estimation results of the FIAPARCH model, we found that the volatility of SSE returns appears to be a long-memory process, in contrast to the assumption of the efficient market hypothesis, and that the volatility of the SSE returns is asymmetric, implying that unexpected negative returns result in more volatility than do unexpected positive returns of the same magnitude.

From the analysis of the in-sample VaR results calculated by the $\text{FIAPARCH}(1, d, 1)$ model, the normal and Student's *t* distribution innovations show poor performance for long and short trading positions. In contrast, the skewed Student's *t* FIAPARCH models significantly improve the in-sample VaR performance for both long and short trading positions. The empirical results for the out-of-sample VaR analysis are found to be similar to those for the in-sample VaR analysis. These results of the in-sample and out-of-sample VaR analyses imply that the FIAPARCH VaR models with the skewed Student's *t* innovation predict critical loss more accurately than do the models with the normal and Student's *t* innovations for both long and short positions.

Overall, we conclude that the density of the SSE returns exhibits skewed and fat-tailed characteristics. Additionally, the volatility displays long-memory and asymmetry features. Therefore, risk managers and portfolio investors can estimate VaR and optimal margin levels most accurately by using the skewed Student's *t* FIAPARCH VaR models of long and short trading positions in the Shanghai stock market.

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