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Multiple Regions Dynamic Stochastic General Equilibrium Model with Oil Production

Bulat Mukhamediyev
Al-Farabi Kazakh National University, School of Economics and Business, Department of Economics, Kazakhstan

Abstract

In today's world there is interdependency between all economies. In particular, all of the economys are affected by fluctuations in oil prices. We have developed multiple regions dynamic stochastic general equilibrium model, which could be considered as the following development of the model of Obstfeld and Rogoff (2001) and its subsequent variants. In the model, some countries produce oil, while others are net importers of it. In each country domestic and foreign goods are consumed. For the production firms employ labor and oil as an energy resource. Price rigidity is modeled by applying the Calvo pricing. Model evaluation and analysis of the impact of internal and external shocks are carried out for the economy of Kazakhstan and its major trading partners.

Keywords: Multiple regions , general equilibrium, dynamic model, macroeconomic shocks

1. Introduction

In today's world all economies are interdependent. In particular, all economies are affected by fluctuations in the price of oil. Here is a model of the dynamic stochastic general equilibrium of several countries, which is an extension of the model for two countries developed by Obstfeld M. and Rogoff K. (2001), Gorsetti C. and Pezenty P. (2001) and subsequent models of Kolasa M. (2008), Gunter U. (2009), Smets F. and Wouters R (2002). Domestic and foreign goods are consumed in each country. Firms employ labor in production. Nominal price rigidity is modeled using the Calvo mechanism.

The bases of the dynamic stochastic general equilibrium models (DSGE) theory have laid by Kydland F.E., Prescott E.C. (1982), who proposed their use to study business cycles. They are based on microeconomic analysis of agents who optimize their behavior under flexible prices. Price flexibility leaves room only for real values to cause fluctuations in the economy. They may be technological shocks or sudden changes in government spending.

Later dynamic general equilibrium models with stochastic shocks were improved. Elements of the Keynesian approach, containing nominal rigidities have been included in the models. In paper of Calvo G. (1983), a pricing mechanism was proposed as a defined stochastic process of decision-making by firms to change the price or keeping it at the same level. Such models are called new Keynesian DSGE models. They take into account the microeconomic foundations of decision-making by households, the optimization behavior of monopolistically competitive firms and regulatory functions of the state. Due to nominal rigidities in prices and wages the required match of calculation results according to the model with real data of short-term macroeconomic fluctuations in the economy is reached.

The principal advantage of models of dynamic stochastic general equilibrium is that they do not fall under the criticism of Lucas R.E. (1976), which is applied to econometric models. For example, a commonly used method of vector autoregression and error correction models, although sometimes prove to be useful, have significant drawbacks (Kumhof M. et al., 2010). They do not take into account inflation expectations, which play a crucial role in the behavior of economic agents.

Among the most famous models created over the past two decades, and designed for policy analysis and forecasting, there are a number of models for the central banks of Europe, Dib A. (2001), Cuche-Curtia N., Dellasb H. and J.-M. Natalc (2009), for developing countries Medina J. and Soto C. (2007), Florian D., Montoro C. (2014), Tovar C. (2008). Other DSGE models were presented in the articles of Galí J. and Monacelli T. (2005), Guerron-Quintana P. (2010), Curdia V. and Woodford M. (2010), Adjemian S. et al. (2009).

2 Model

In each country there are consumed domestic and foreign goods. For the production of the company employ labor. Nominal price rigidity is modeled with the use of Calvo price mechanism. It is believed that the world's population is consists of a continuum of infinitely long-lived households indexed by i∈ [0,1]. Households in each country have the same preferences. There are K countries in the world. The country k households are indexed as $i \in J_k$. The sets $J_k,\ \ k = 1,\ 2, \ldots, K$, do not intersect and cover the entire set of households. Through n_k we denote the measure of J_k, which reflects the measure of population of the country k.

2.1 Consumption

In the country k the composite consumption index is determined by assuming that all the goods are traded and trading costs are ignored:

$$C_k = \left(\sum_{l=1}^{k} n_l^{\frac{1}{\mu}} \, C_{kl}^{\frac{\mu-1}{\mu}} \right)^{\frac{\mu}{\mu-1}} \tag{1}$$

where C_{kl} is a composite index of consumer goods in the country k, produced in the country l, μ is a parameter. A representative household maximizes C_k under the restriction

$$\sum_{l=1}^{k} P_{kl}\, C_{kl} = P_k\, C_k \ , \tag{2}$$

where P_{kl} is price index of goods in a country l expressed in the currency of a country k, P_k is the index of prices of all goods consumed in the country k. We write the necessary condition for the maximization problem (1) subject to (2) and after transformations we obtain that the composite index of consumption of goods in the country k, produced in the country l

$$C_{kl} = \left(\sum_{j=1}^{k} n_j \; P_{kj}^{1-\mu}\right)^{\frac{\mu}{1-\mu}} \frac{n_l}{P_{kl}^{\mu}} \, C_k \, , \tag{3}$$

and the consumer price index in the country k is

$$P_k = \left(\sum_{l=1}^{k} n_l \; P_{kl}^{1-\mu}\right)^{\frac{1}{1-\mu}} . \tag{4}$$

In the limit by μ, tending to 1, we find that the price index is

$$P_k = \prod_{l=1}^{K} P_{kl}^{n_l}. \tag{5}$$

Now, by substituting the expression (4) in the equation (3), we find a formula for the index of consumption of goods from the country l to the country k:

$$C_{kl} = n_l \left(\frac{P_{kl}}{P_k}\right)^{-\mu} C_k, \quad k, l = 1,2,\ldots,K. \tag{6}$$

In the limit $\mu \to 1$ the composite consumption index (1) takes the form of a power function

$$\mathrm{C}_k = \prod_{l=1}^{K} \frac{C_{kl}^{n_l}}{{n_l}^{n_l}} = \frac{C_{k1}^{n_1} \, C_{k2}^{n_2} \, \ldots \, C_{kK}^{n_K}}{n_1^{n_1} \, n_2^{n_2} \, \ldots \, n_K^{n_K}}. \tag{7}$$

Then equation (6) can be written as

$$C_{kl} = n_l \left(\frac{P_{kl}}{P_k}\right)^{-1} C_k, \quad k, l = 1, \ldots, K. \tag{8}$$

Here and everywhere below the index for period t is omitted if it is not essential. The index of consumption of goods in the country k, produced in the country l:

$$C_{kl} = \left[\left(\frac{1}{n_l}\right)^{\frac{1}{\eta}} \int_J C_k(i)^{\frac{\eta-1}{\eta}} \, di\right]^{\frac{\eta}{\eta-1}}. \tag{9}$$

where $C_k(i)$ is the consumption of a good i in the country k. A representative household maximizes C_{kl} on $C_k(i), i \in J_l$ subject to

$$P_{kl}C_{kl} = \int_{J_l} P_k(i)C_k(i)di, \quad (10)$$

where $P_k(i)$ is the price of a good *i* in country *k*, and P_{kl} is the price index of goods from *l* to the country *k*. A first-order optimality condition after transformation leads to the formula

$$C_k(i) = \frac{1}{n_l}\left[\frac{P_k(i)}{P_{kl}}\right]^{-\eta} C_{kl} \quad (11)$$

for *i*=J_l, *k,l*=1,2,…,K. A world consumption of good i, following Obstfeld M. and Rogo ff K. (2001), is written in the form

$$C^w(i) = \sum_{k=1}^{K} n_k C_k(i).$$

We take into account that $P_k(i) = \mathcal{E}_{kl}\, P_l(i)$, $P_{kl} = \mathcal{E}_{kl}\, P_{ll}$, $P_k = \mathcal{E}_{kl}\, P_l$, where $\mathcal{E}_{kl}$ is the nominal exchange rate of between countries *k* and *l*, the number of currency units of the country *k* for the unit of currency of the country *l*. Then

$$C^w(i) = \left[\frac{P_l(i)}{P_{ll}}\right]^{-\eta}\left(\frac{P_{ll}}{P_l}\right)^{-\mu} C^w. \quad (12)$$

for a good $i \epsilon J_l$, produced in the country *l*. Here C^w is the global index of consumption of all goods.

2.2 Households

In the country *k* a representative household has discounted utility

$$U_{kt} = E_t\left\{\sum_{s=t}^{\infty} \beta^{s-t}\left[\frac{C_{ks}^{1-\rho}}{1-\rho} + \frac{\omega_k}{1-\delta}\left(\frac{M_{ks}}{P_{ks}}\right)^{1-\delta} - \vartheta_k \frac{L_{ks}^{1+\varphi}}{1+\varphi}\right]\right\}, \quad (13)$$

where C_{ks} is a real consumption, $\frac{M_{ks}}{P_{ks}}$ are real money balances, P_{ks} is a consumer price index in country *k*, L_{ks} are labor costs in time period s. The parameter β, 0 <β <1, is an intertemporal discount factor, the parameters ρ, δ, φ determine the elasticity of the utility function of the corresponding variables. Representative household (i) maximizes the utility of (13) with budget constraints

$$P_{kt}C_{kt} + M_{kt} + B_{kt} + P_{kt}\tau_{kt} \le W_{kt}L_{kt} + (1 + i_{kt-1})B_{kt-1} + M_{kt-1} + \Pi_{kt}. \quad (14)$$

Here, for the country k and period t there following are marked: W_{kt} – a nominal wage in perfect labor market, the same for all households, i_{kt-1} is the nominal interest rate for the time interval from t-1 to t for one-period risk-free corporate bonds B_{kt-1} in the domestic currency. Money M_{kt} do not give nominal income. Π_{kt} is an income of a representative household, τ are real undistorted lump sum taxes. The index of a household $i \epsilon J_k$ is omitted for simplicity. Using the Lagrangian function for the problem of discounted utility maximization of the household (13) under the constraints (14), we obtain the optimality conditions:

$$\frac{C_{kt}^{-\rho}}{P_{kt}} = \beta(1+i_{kt})E_t\left[\frac{C_{kt+1}^{-\rho}}{P_{kt+1}}\right], \quad \vartheta_k \frac{L_{kt}^{\varphi}}{C_{kt}^{-\rho}} = \frac{W_{kt}}{P_{kt}}, \quad \frac{\omega\left(\frac{M_{kt}}{P_{kt}}\right)^{-\delta}}{C_{kt}^{-\rho}} = \frac{i_{kt}}{1+i_{kt}}. \tag{15}$$

The conditions (15) are valid for each country *k = 1, ..., K.*

2.3 Firms

It is assumed that each household is also the producer of the product $i \in J_k$. Goods are considered diversified, therefore, each such firm has a market power. In the simplest version output of each firm $i \in J_k$ is determined by the production function

$$Y_{kt}(i) = A_{kt}\, L_{kt}(i). \tag{16}$$

The value A_{kt} sets the impact of the productivity shock. It is assumed that in different countries, these values can be correlated. Here A_{kt} reflects technological shocks. The behavior of A_{kt} is described by autoregressive process

$$lnA_{kt} = \rho_{ak} lnA_{kt-1} + \varepsilon_{akt}, \quad \varepsilon_{akt} \sim i.i.d.(0, \sigma_{ak}^2).$$

Since goods are expected to be diversified a firm can within certain limits change the price of a good, i.e., there is a monopolistic competition. Labor markets are isolated. Firms hire labor in the country. For the production function a firm's profit

$$\Pi_{kt}(i) = P_{kt}(i)Y_{kt}(i) - W_{kt}L_{kt}(i). \tag{17}$$

A value

$$MC_{kt} = \frac{W_{kt}}{A_{kt}P_{kt}(i)}$$

is the real marginal costs of the firm. The optimal output of a good $i \in J_k$ is determined by maximizing $\Pi_{kt}(i)$ on $Y_{kt}(i)$. Let's consider the case of flexible prices, i.e. all firms in each period t optimally adjust their prices. Then all the producers set the same prices, $P_{kt}(i) = P_{kt}$. Since only the firm *i* produces this product, the equilibrium output must be equal to the global demand for it, that is $Y_{kt}(i) = C_t^w(i)$. Note that

$$\frac{Y_{kt}(i)}{P_{kt}} \frac{\partial P_{kt}}{\partial Y_{kt}(i)} = \frac{C_t^w(i)}{P_{kt}} \frac{\partial P_{kt}}{\partial C_t^w(i)} = -\frac{1}{\eta},$$

where η is elasticity of demand on price. Consequently, the real marginal costs of production in the case of flexible prices are the same for all manufacturers in all countries: $\widetilde{MC} = \frac{\eta-1}{\eta}$.

2.4 Equilibrium at flexible prices

A condition of market clearing of a good i is expressed in the equality of supply volume of this product to the total demand for all countries:

$$Y_{kt}(i) = \sum_{l=1}^{K} n_l C_{lt}(i), \qquad i \in J_k, \qquad k = 1, \dots, K.$$

It is believed that consumption is distributed across countries in proportion to the population. Through S_{klt} we denote the terms of trade between countries k and l:

$$S_{klt} = \frac{P_{klt}}{P_{kkt}}.$$

It is assumed that the government supports a non-deficit state budget, i.e., $M_{kt} = M_{kt-1} - P_{kt}\tau_{kt}$. Then the budget constraint (14) of the representative household in the country k can be written as

$$i_{kt-1} B_{kt-1} + P_{kkt} Y_{kt} - P_{kt} C_{kt} = B_{kt} - B_{kt-1}, \qquad k = 1, \dots, K \tag{18}$$

Securities are considered traded between countries, and

$$\sum_{k=1}^{K} n_k B_{kt} = 0.$$

If B_{kt} is positive, the country k is a net lender, and if B_{kt} is negative, it acts on the world market as a net borrower. Position in the bond market is determined according to Obstfeld M. and Rogoff K. (2001, p. 10) by the equality $B_{kt} = P_{kkt}Y_{kt} - P_{kt} C_{kt}$. Assume that the initial position of the bonds are zero, i.e., $B_{k0} = 0,\ k = 1, \dots, K.$ According to Gorsetti C. and Pezenty P. (2001), Obstfeld M. and Rogoff K. (2001) in these conditions the following relations are hold:

$$C_{kt} = \frac{P_{kkt} Y_{kt}}{P_{kt}}, k = 1, \dots, K. \tag{19}$$

They mean that each country consumes exactly its real income. Hence

$$C_{kt} = \frac{P_{kkt}Y_{kt}}{\prod_{l=1}^{K} S_{klt}^{n_l}} = \prod_{l=1}^{K} S_{klt}^{-\eta}\, Y_{kt}. \tag{20}$$

Assume that $S_{kk} = 1$. From (19) it follows that $P_{kt}C_{kt} = P_{kkt}Y_{kt} = P_{kt}C_t^w$, i.e. $C_{kt} = C_t^w$, $k = 1, \dots, K$. Then

$$C_t^w = \sum_{l=1}^{K} n_l C_{lt} = \sum_{l=1}^{K} n_l \prod_{m=1}^{K} S_{lmt}^{-n_m} Y_{lt}\,.$$

In the optimality condition (15) we replace M_{kt} on $M_{kt-1} - P_{kt}\tau_{kt}$ in accordance with the condition of deficit-free of a state budget. Taking into account (19) after transformations we obtain

$$P_{kt} = \frac{M_{kt-1}}{w_k^{\frac{1}{\delta}} \left(\frac{1+i_t}{i_t}\right)^{\frac{1}{\delta}} \left(\prod_{l=1}^{K} S_{klt}^{-n_l}\, Y_{kt}\right)^{\frac{\rho}{\delta}} + \tau_t}\,.$$

Let’s write a similar expression for P_{mt} and given that $P_{kt} = \mathcal{E}_{km}P_{mt}$, we find that

$$\mathcal{E}_{kl} = \frac{M_{kt-1}\left[w_m^{\frac{1}{\delta}} \left(\frac{1+i_{mt}}{i_{mt}}\right)^{\frac{1}{\delta}} \left(\prod_{l=1}^{K} S_{mlt}^{-n_l}\, Y_{mt}\right)^{\frac{\rho}{\delta}} + \tau_{mt}\right]}{M_{mt-1}\left[w_k^{\frac{1}{\delta}} \left(\frac{1+i_{kt}}{i_{kt}}\right)^{\frac{1}{\delta}} \left(\prod_{l=1}^{K} S_{klt}^{-n_l}\, Y_{kt}\right)^{\frac{\rho}{\delta}} + \tau_{kt}\right]}.$$

This formula shows the dependence of the nominal exchange rate form the money supply, the interest rates, the output of countries k and m, as well as the terms of trade of these countries with other countries. Let’s turn back to the labor market. Let’s find an optimality condition for labor(15) in terms of L_{kt}^{φ}. By substituting in it the expression for C_{kt} from (20), then by applying the formula (5) we obtain

$$L_{kt} = \left(\frac{A_{kt}}{\chi}\right)^{\frac{1}{\varphi}} \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\varphi}} \prod_{l=1}^{K}\left[S_{klt}^{\frac{\rho(n_l-1)}{\varphi}}\right] Y_{kt}^{\frac{-\rho}{\varphi}} \tag{21}$$

In the country in a perfectly competitive labor market employment equilibrium depends positively on productivity, real marginal costs with flexible prices MC_{kt} and negatively on the terms of trade of the country with other countries. Now, using the production function (16), we can calculate the equilibrium output under flexible prices.

$$\tilde{Y}_{kl} = A_{kt}^{\frac{\varphi+1}{\varphi+\rho}}\, \chi^{\frac{-1}{\varphi+\rho}} \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\varphi+\rho}} \prod_{l=1}^{K}\left[S_{klt}^{\frac{\rho(n_l-1)}{\varphi+\rho}}\right] \tag{22}$$

It depends positively on the total productivity and negatively on the country's terms of trade with other countries as $n_l < 1$.

2.5 Dynamic IS curve

Suppose now that in addition to the monopolistic competition nominal price rigidity is present. So, as done in many works on dynamic stochastic equilibrium, a pricing mechanism proposed by Calvo (1983) is applied here. For the country k in the first Euler equation (15) we substitute the real consumption C_{kt} from equation (20):

$$\prod_{l=1}^{K}\left(S_{klt}^{\rho n_l}\right)Y_{kt}^{-\rho} = \beta(1+i_{kt})P_{kt}\,E_t\left[\frac{1}{P_{kt+1}}\prod_{l=1}^{K}\left(S_{klt+1}^{\rho n_l}\right)Y_{kt+1}^{-\rho}\right].$$

At a steady state of economy the output is denoted by $\bar{Y}_k$, through $\bar{S}_{kl}$ – terms of trade of country k with the country l, $\bar{\iota}_k$ – a nominal interest rate, $\bar{P}_k$ – a price index of goods in the country k are denoted. Let's write an equation for the steady state of the economy and find the ratio of the left and right sides of these two equations. Then log-linearization of this equation is applied. Let's take logarithm of both sides and denote: $s_{klt} = \ln S_{klt}$, $y_{kt} = \ln Y_{kt}$, $\bar{s}_{kl} = \ln \bar{S}_{kl}$, $\bar{y}_k = \ln \bar{Y}_k$, $p_{kt} = \ln P_{kt}$, $\bar{p}_k = \ln \bar{P}_k$. Using the properties of logarithms, we obtain the equation:

$$\sum_{l=1}^{K}\rho n_l s_{klt} - \rho y_{kt} - \sum_{l=1}^{K}\rho n_l \bar{s}_{kl} + \rho\bar{y}_k = \ln(1+i_{kt}) -$$

$$-\ln(1+\bar{\iota}_k) + P_{kt} - E[p_{kt+1}] + \sum_{l=1}^{K}\rho n_l E_t[s_{klt}] - \rho E_t[y_{kt+1}] - \sum_{l=1}^{K}\rho n_l\, E_t[\bar{s}_{kl}] + \rho\bar{y}_k\,.$$

Denote deviations of the variables from their values in the steady state: $\hat{y}_{kt} = y_{kt} - \bar{y}_k$, $\hat{\iota}_{kt} = i_{kt} - \bar{\iota}_k$. Then the equation is transformed into:

$$-\rho\hat{y}_{kt} = -\rho E_t[\hat{y}_{kt+1}] - E_t[p_{kt+1} - p_{kt}] + \hat{\iota}_{kt} + \rho\sum_{l=1}^{K} n_l E_t[s_{klt+1} - s_{klt}].$$

Note that the difference $\pi_{kt+1} = p_{kt+1} - p_{kt}$ represents the rate of inflation in period t + 1. The equation can be written as:

$$\hat{y}_{kt} = E_t[\hat{y}_{kt+1}] + \frac{1}{\rho}E_t[\pi_{kt+1} - \hat{\iota}_{kt}] - \sum_{l=1}^{K} n_l E_t\,[\Delta s_{klt+1}], \qquad k = 1,,,K. \tag{23}$$

This is the equation of dynamic IS curve. It sets the aggregate demand in the country k. In period t aggregate demand will increase if the expected output in period t + 1 will be higher than its steady state. Expectation of inflation growth will also increase the current demand for domestic goods. And the expected improvement in the terms of trade with the other countries, i.e., positive value of $\Delta\, s_{klt+1}$ will reduce the current aggregate demand, as prices of imported goods become relatively higher than prices of domestic goods.

2.6 New Keynesian Phillipps curve

In accordance with the mechanism of Calvo price correction a producer i change the price in each period with probability 1-θ, maximizing the expected profit at the price $P_t(i)$:

$$E_t\left\{\sum_{s=t}^{\infty}\theta^{s-t}\beta^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}\left[\frac{P_{kt}(i)}{P_{kks}}Y_{ks}(i)-MC_{ks}Y_{ks}(i)\right]\right\}.$$

Here $\beta^{s-t}\left(\frac{C_s^w}{C_t^w}\right)^{-\rho}$ is the stochastic discount factor, which is the marginal rate of substitution in global consumption between periods of s and t, MC_{ks} are marginal costs of production in the country k at period s. With probability θ^{s-t} a producer's price in the period s> t is equal to $P_{kt}(i),\ i\epsilon J_k$. The profit of the firm in the period s, which set the price in period t, is equal to:

$$\Pi_{ks}(i)=P_{kt}(i)Y_{ks}(i)-W_{ks}\frac{P_{kks}}{A_{ks}P_{kks}}Y_{ks}(i)=P_{kt}(i)\,Y_{ks}(i)-MC_{ks}Y_{ks}(i)P_{kks}.$$

We divide the price P_{kks} and find a real profit of the firm $i\epsilon J_k$ in the period $s\ (s\geq t)$.

$$\frac{\Pi_{ks}(i)}{P_{kks}}=\frac{P_{kt}(i)}{P_{kks}}Y_{ks}(i)-MC_{ks}Y_{ks}(i).$$

Substituting the expression (12) into the objective function of the firm instead $Y_{ks}(i)$. Let's write the necessary condition for the maximum of this function on $P_{kt}(i)$. After some transformations we obtain:

$$P_{kt}(i)\sum_{s=t}^{\infty}(\theta\beta)^{s-t}E_t\left[MC_{ks}\left(\frac{P_{kks}}{P_{kkt}}\right)^{\eta}\left(\frac{P_{kks}}{P_{ks}}\right)^{-1}\mathrm{C}_s^{\mathrm{w}1-\rho}\right]=$$
$$=P_{kkt}\frac{\eta-1}{\eta}\sum_{s=t}^{\infty}(\theta\beta)^{s-t}\ E_t\left[\left(\frac{P_{kks}}{P_{kkt}}\right)^{\eta-1}\left(\frac{P_{kks}}{P_{ks}}\right)^{-1}\mathrm{C}_s^{\mathrm{w}1-\rho}\right].\qquad(24)$$

In the case of flexible prices all firms change prices in each period, i.e. θ = 0. Then,

$$P_{kt}(i)=\frac{\eta}{\eta-1}MC_{kt}P_{kkt}.$$

Since prices $P_{kt}(i)$ coincide with P_{kkt}, then $1=\frac{\eta-1}{\eta}MC_{kt}$, i.e. again we obtain

$$\widetilde{MC}_{kt}=\frac{\eta-1}{\eta}=\widetilde{MC}.$$

Next let prices to be rigid, which corresponds to a positive value θ. Draw log-linearization of equation (24).

$$\hat{p}_{kt}(i) - \hat{p}_{kkt} = (1-\theta\beta)\widehat{mc}_{kt} + \theta\beta\, E_t\, [\hat{p}_{kt+1} - \hat{p}_{kkt+1} + \hat{\pi}_{kkt+1}]. \quad (25)$$

By transformations we obtain the equation

$$\hat{p}_{kt}(i) = \frac{1}{1-\theta}\hat{p}_{kkt} - \frac{\theta}{1-\theta}\hat{p}_{kkt-1}.$$

We substitute this expression in the equation (25) and obtain:

$$\pi_{kkt} = \beta E_t\, [\pi_{kkt+1}] + \frac{(1-\theta\beta)(1-\theta)}{\theta}\, \widehat{mc}_t. \quad (26)$$

This is new Keynesian Phillips curve for country k. Here $\widehat{mc}_t = mc_t - \widetilde{mc}_t$. Note the difference in determining the rate of inflation in the equation Phillipps and in the equation of dynamic curve IS. In equation (24) π_{kt} determined by the consumer price Index, as in equation (26) π_{kkt} is the growth rate of prices of goods produced in country k. We turn to the deviations of output under rigid prices from the output under flexible prices: $x_{kt} = \hat{y}_{kt} - \hat{\tilde{y}}_{kt}$. Transform second equation (15) for the case of flexible prices, using equations (16) and (20).

$$\vartheta_k \left(\frac{Y_{kt}}{A_{kt}}\right)^{\varphi} Y_{kt}^{\rho} = \frac{\eta - 1}{\eta}\, A_{kt} \prod_{l=1}^{K} S_{klt}^{(\rho-1)n_l}.$$

Hence, the output under flexible prices is:

$$\tilde{Y}_{kt}^{\varphi+\rho} = \frac{1}{\vartheta_k}\,\frac{\eta-1}{\eta}\, A_{kt}^{\varphi+1} \prod_{l=1}^{K} S_{klt}^{(\rho-1)n_l}. \quad (27)$$

And after log-linearization we obtain

$$\widehat{mc}_{kt} = (\varphi+\rho)(y_{kt} - \hat{y}_{kt}) = (\varphi+\rho)x_{kt}. \quad (28)$$

Rewrite the equation (23) of dynamic IS curve using the deviation of output at rigid prices from output under flexible prices:

$$x_{kt} = E_t[x_{kt+1}] + \frac{1}{\rho}\, (E_t\, [\pi_{kt+1}] - \hat{\imath}_{kt}) - \sum_{l=1}^{K} n_l\, E_t[\Delta s_{klt+1}] - \hat{\tilde{y}}_{kt} + E_t\big[\hat{\tilde{y}}_{kt+1}\big] \quad (29)$$

And the equation (26) of neoclassical Phillips curve:

$$\pi_{kkt} = \beta E_t[\pi_{kkt+1}] + \frac{(1-\theta\beta)(1-\theta)}{\theta}\, (\varphi+\rho)x_t + u_{kt}, \quad (30)$$

where u_{kt} is an autoregressive process.

$$u_{kt} = \rho_{uk} u_{kt-1} + \upsilon_{kt}, \quad \upsilon_{kt} \sim iid\ (0, \sigma_{\upsilon k}^2).$$

By log-linearization out of (27) we obtain the following for flexible prices

$$\tilde{y}_{kt} = -\frac{1}{\varphi+\rho} \ln \chi + \frac{1}{\varphi+\rho} \ln\left(\frac{\eta-1}{\eta}\right) + \frac{\varphi+1}{\varphi+\rho} a_{kt} + \frac{\rho-1}{\varphi+\rho} \sum_{l=1}^{K} n_l s_{klt}.$$

Let's calculate the sum of the last two terms in the equation (29).

$$E_t\left[\hat{\hat{y}}_{kt+1}\right] - \hat{\hat{y}}_{kt} = E_t\left[\hat{y}_{kt+1}\right] - \hat{y}_{kt} = \frac{\varphi+1}{\varphi+\rho} E_t\left[\Delta a_{kt+1}\right] + \frac{\rho-1}{\varphi+\rho} \sum_{l=1}^{K} n_l\, E_t[\Delta s_{klt+1}].$$

By definition, the k country's terms of trade with the country l are :

$$S_{klt} = \frac{\xi_{klt} P_{lkt}}{P_{kkt}} = \frac{P_{klt}}{P_{kkt}}.$$

Taking the logarithm of both sides we get:

$$s_{klt} = e_{klt} + p_{lkt} - p_{kkt} = p_{klt} - p_{kkt}.$$

Hence we have:

$$E_t\left[\Delta s_{klt+1}\right] = E_t\left[\pi_{klt+1}\right] - E_t[\pi_{kkt+1}].$$

The equation follows from the formula (4):

$$E_t\left[\pi_{kt+1}\right] = E_t[\pi_{kkt+1}] + \sum_{l=1}^{K} n_l\, E_t[\Delta s_{klt+1}].$$

The equation of the dynamic IS curve can also be rewritten as:

$$x_{kt} = E_t[x_{kt+1}] + \frac{1}{\rho}\left(E_t\left[\pi_{kkt+1}\right] - \hat{\imath}_{kt}\right) + \frac{\varphi+1}{\varphi+\rho} E_t\left[\Delta a_{kt+1}\right] + \\ + \frac{\varphi(1-\rho)}{\rho(\varphi+\rho)} \sum_{l=1}^{K} n_l E_t[\Delta s_{klt+1}]. \quad (31)$$

To exclude currency speculations the uncovered interest arbitrage conditions must be satisfied:

$$1 + i_{kt} = (1 + i_{lt}) \frac{E_t\,[\mathcal{E}_{klt+1}]}{\mathcal{E}_{klt}}, \qquad k \neq l.$$

Using the log-linearization based on the terms of trade we obtain the equation:

$$\Delta s_{klt} = \hat{\imath}_{kt-1} - \hat{\imath}_{lt-1} + \pi_{llt} - \pi_{kkt}, \quad k \neq l. \tag{32}$$

Note that $\Delta s_{lkt} = -\Delta s_{klt}$, and the relationship between increments of logarithms of terms of trade between countries $\Delta s_{lmt} = \Delta s_{kmt} - \Delta s_{klt}$. Consequently, there are K-1 independent values Δs_{klt} , for example, Δs_{1lt}, l =2, …, K. The remaining values Δs_{mlt} are expressed through them.

To equations (30) - (32) the equations determining the interest rate movements should be added. According to the Taylor rule of monetary policy (1993) interest rates are set by central banks in accordance with the formula of the following form:

$$i_{kt} = \psi_{\pi k}\pi_{kkt} + \psi_{xt}x_{kt} + \psi_{ik}\hat{\imath}_{kt-1} + \upsilon_{kt}, \quad k = 1, \dots, K. \tag{33}$$

It is assumed that the υ_{kt} dynamics is determined exogenously by an autoregressive process of the first order:

$$\upsilon_{kt} = \rho_\upsilon \upsilon_{kt-1} + \upsilon_{\upsilon kt,} \quad \upsilon_{k\upsilon t} \sim iid\,(0, \sigma_{k\upsilon}^2).$$

Thus, the model of K countries is described by 4K-1 equations (30)-(33) and contains the same number of variables, if the connections (32) between increments of logarithms of the terms of trade are taken into account. Statistical data to build the model of dynamic stochastic equilibrium for the three countries have been collected according to the International Financial Statistics of International Monetary Fund, the World Bank, the Agency for Statistics of Kazakhstan, and the National Bank of Kazakhstan.

3 Model of dynamic stochastic general equilibrium of the three countries

The developed model for several countries we specify for the case of three countries (regions). The model was evaluated according using statistical data of Kazakhstan (country H), Russia (Country F), and European Union (country G), the International Monetary Fund, the World Bank, the Agency for Statistics, and the National Bank of Kazakhstan. In particular, according to the proportion of the population, the following values are taken:

$$n_H = 0.03, \;\; n_F = 0.22, \;\; n_G = 0.75.$$

Indeed, Kazakhstan is a relatively small country. Figures 1-2 shows how shocks in each country react to macroeconomic indicators in this country and in other countries. A technological shock in Kazakhstan initially reduces the output in Kazakhstan and increases the outputs in Russia and European Union (Figure 1). The same effect incurs on the rate of inflation. The interest rates in Kazakhstan decreases, and vice versa in Russia and European Union they increase. However, we note that the effect of this shock in Russia and European

Union are of orders with magnitude weaker than in Kazakhstan.

The situation is similar in the case of technology shock in Russia. But the scale is different. The impact of technological shocks on macroeconomic indicators in Russia and Kazakhstan are of the same order, and in European Union are lower on three orders of magnitude.

As shown in Figure 2, the response graphics of macroeconomic indicators for technology shock in European Union are similar to the graphs of the impulse functions of Kazakhstan and Russia, but unlike them their responses in all three countries have the same order. There the size of the country G with respect to Kazakhstan and Russia plays an important role.

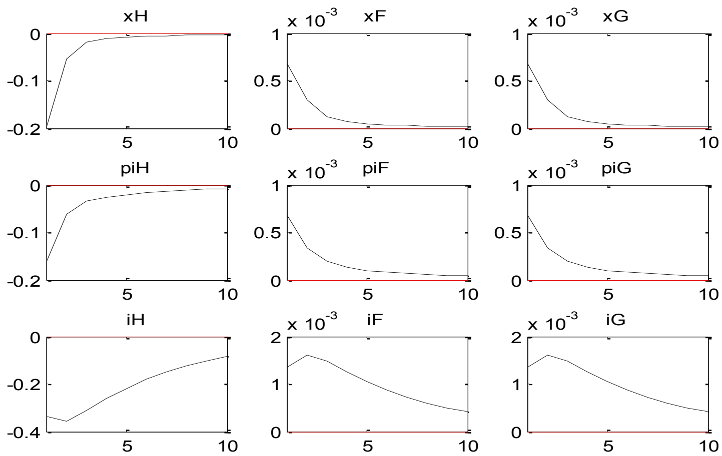

Figure 1 – The impact of technological shock in the country H (Kazakhstan)

Note - xH, xF, xG - are the deviations of output at rigid prices to output under flexible prices in logarithms, piH, piF, piG are inflation measured by producer price indecies, iH, iF, iG are interest rate deviations from their values at steady state for countries H (Kazakhstan), F (Russia), G (European Union), respectively.

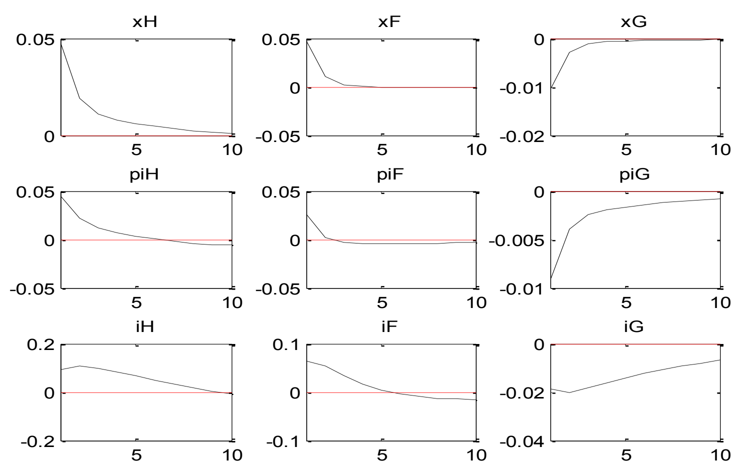

Figure 1 – The impact of technological shock in the country G (European Union)

Note - xH, xF, xG - are the deviations of output at rigid prices to output under flexible prices in logarithms, piH, piF, piG are inflation measured by producer price indecies, iH, iF, iG are interest rate deviations from their values at steady state for countries H (Kazakhstan), F (Russia), G (European Union), respectively.

Effect of production costs shock can also be represented in the figures. In a country in which such a shock occurs, there is a decline in production, rising inflation and interest rates, and in other countries also a decline in production and a decrease in inflation and interest rates are observed. But the consequences of the shock of production costs in Kazakhstan to four orders of magnitude are weaker in Russia and European Union, than in Kazakhstan. The effects of such a shock in Russia for Kazakhstan and Russia the country of the same order, however, they are weaker for four orders of magnitude than for European Union.
A response shock of production costs in European Union for Kazakhstan and Russia are three orders weaker than for European Union. Here there are noticeable differences in production costs shock effects from the effects of the technological shocks. It is transferred substantially weaker than in other countries in comparison with the technological shock. Monetary shock leads to a decrease in output, lowering of inflation and interest rates both in the country and in other countries. But in those other countries the impact is three to four times less than in the country-origin of the shock.

4 Model with the production of oil

Let country k besides the production of final goods is producing oil. It has oil sector with production function:

$$O_{kt} = A_{okt} L_{okt}^{\nu}\ ,\ \ 0 < \nu < 1,$$

where O_{skt} is oil supply, , L_{okt} is number of employees in the oil sector, A_{okt} is factor productivity in the production of oil, reflecting the impact of technological shocks,

$$lnA_{okt} = \rho_{ok} lnA_{okt-1} + \varepsilon_{okt},\quad \varepsilon_{okt} \sim\ i.i.d.(0, \sigma_{ok}^2).$$

Taking into account energy costs the production function of the firm that produces the final product i, in contrast to (16) has the form

$$Y_{lt}(i) =\ A_{lt}\ min\ \left\{L_{lt}(i), \frac{1}{\zeta} O_t(i)\right\}, i \in J_l,$$

where $O_{lt}(i)$ are the costs of oil as an energy resource, $L_{lt}(i)$ are the costs of labor, ζ is a parameter of energy consumption in the production of final goods. For simplicity, we consider here only the case when oil is produced only in the country k. Total labor supply L_{kt} in country k is divided into involved in the production of oil L_{okt} and engaged in the production of final goods $L_{kt}(i),\ i \in J_k$, as well as the supply of oil is divided by its quantity O_{lt} in the countries l = 1,2 , ..., K, respectively, i.e.

$$L_{kt} = L_{okt} + \int_{J_k} L_{kt}(i)di, \quad O_{kt} = \sum_{l=1}^{K} O_{lt}, \quad O_{lt} = \int_{J_l} O_{lt}(i)di, \quad l = 1,2,\dots,K.$$

Since the production of goods and energy costs are taken into account, it affects the marginal costs of the firm:

$$MC_{lt} = \frac{W_{lt}+\zeta\mathcal{E}_{lmt}P_{ot}}{A_{lt}P_{lt}(i)}, \ l = 1,2,\dots,K,$$

where P_{ot} – world oil price, expressed in the currency of country m. Oil costs affect the profit of the firm

$$\Pi_{lt}(i) = P_{lt}(i)Y_{lt}(i) - W_{lt}L_{lt}(i) - \mathcal{E}_{lmt}P_{ot}O_{lt}(i).$$

In the country k oil sector will have the profit

$$\Pi_{ot} = \mathcal{E}_{kmt}P_{ot}O_{kt} - W_{kt}L_{okt}.$$

Consequently budgetary constraints of representative households will change in the country l, l ≠ k:

$$P_{lt}C_{lt} + M_{lt} + B_{lt} + P_{lt}\tau_{lt} \le W_{lt}L_{lt} + (1 + i_{lt-1})B_{lt-1} + M_{lt-1} + P_{lt}Y_{lt} - W_{lt}L_{lt} - \mathcal{E}_{lmt}P_{ot}O_{lt},$$

and in the country k:

$$P_{kt}C_{kt} + M_{kt} + B_{kt} + P_{kt}\tau_{kt} \le W_{kt}L_{kt} + (1 + i_{kt-1})B_{kt-1} + M_{kt-1} + P_{lt}Y_{lt} - W_{kt}L_{kt} -$$
$$-\mathcal{E}_{kmt}P_{ot}O_{kt} + \mathcal{E}_{kmt}P_{ot}O_{kt} - W_{kt}L_{okt}.$$

Establishment of the dynamic IS curve and Neo-Keynesian Phillips curve is similar to the method, used for the model of an open economy (Mukhamediyev B., 2013). However, these changes in the model associated with the account of production and consumption of oil, greatly complicate mathematical calculations. Therefore, they are not presented here. Further development of the model can be derived by considering oil production in several countries and determining the equilibrium on the base of the games theory.

5 Conclusion

The paper presents a model of dynamic stochastic general equilibrium for several countries, its and its mathematical justification is carried out. This model is evaluated for the case of three countries (regions), according to the statistics of Kazakhstan, Russia and the European Union for the 1995-2012 years. Used data from IFS International Monetary Fund, the World Bank, the Agency of Statistics of Kazakhstan, the National Bank of Kazakhstan. A part of the model parameters were estimated using Bayesian approach and Metropolis-Hastings algorithm. A simulation of this model on the various options of macroeconomic policies in these countries was performed. The impact of technology shocks, shocks in production costs, monetary shocks in the country and in other countries on the dynamics of macroeconomic variables have been considered. As might be expected, the magnitude of responses to shocks substantially depends on the relative sizes of countries. Also a model that takes into account the production and consumption of oil was presented, the definition of equilibrium in which, in general, can be obtained on the basis of the games theory approach.

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